Upon satisfaction of these conditions, the analogy under consideration affords a possibility of analyzing convective thermal and mass transfer with volume sources on the basis of tests on gradient flows.

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HEAT TRANSFER DURING MIXED CONVECTION ON A VERTICAL SURFACE IN A POROUS MEDIUM WITH DEVIATION FROM DARCY'S LAW

A. V. Gorin, V. E. Nakoryakov,
A. G. Khoruzhenko, and O. N. Tsoi

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In recent years, the requirements of modern technology have stimulated interest in the study of flows which involve the interaction of several phenomena. One such problem is heat transfer during mixed natural and forced convection in porous media. The need to solve these problems stems from the broad use of granular media in chemical engineering (granular beds of catalysts) and the use of geothermal power energy sources and methods of intensifying oil and gas extraction, which are based either on organizing a moving combustion source or pumping hot water or steam. The problems are also encountered in the use of heat pipes and other devices.

In these cases, convective heat flows are realized in porous media when a heated (or cooled) object is placed in a fluid whose density changes with temperature. Forced convection occurs when an external flow moves around a surface.

Problems of heat transfer with free and forced convection in a Darcy's law approximation have been studied in the greatest detail to date. Investigators have examined heating surfaces with different geometries (plate, cylinder, flow along the inside surface of a cylinder) and different orientations in space – vertical, horizontal, and inclined plates. A detailed survey of the problems studied is offered in [1]. The problems were solved in a boundary-layer approximation and are based on the Darcy flow model. Conditions were established for the existence of similarity solutions for corresponding methods of assigning boundary conditions, and relations were found for the exponents in power laws describing the distributions of the external flow and wall temperature.

However, we should point out the rather narrow range of applicability of Darcy's law [2]. It is restricted to the Reynolds number limit $Re = u\sqrt{\Pi}/\nu \leq O(1)$, constructed from the

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filtration velocity u and the scale of the pores — proportional to $\sqrt{\Pi}$ (Π is permeability and ν is kinematic viscosity). The nonlinear filtration regime is of the greatest practical interest. Also, this regime becomes additionally important when the boundary-layer approximation is used. Specifically (as noted in [3]), in any finite porous system, the boundary-layer approximation is valid for sufficiently large Rayleigh numbers Ra = $g\beta\Delta TL^3/(\nu a)$ (g is acceleration due to gravity, β is the coefficient of volume expansion, L is a characteristic length, a is diffusivity, and $\Delta T = T_w - T_\infty$; here and below, the indices w and ∞ denote parameters on the plate and at infinity, respectively; T is temperature). This leads to deviation from Darcy's law, since the scale of velocity increases with an increase in Ra, i.e., the Darcy approximation becomes less accurate as the boundary-layer approximation improves in accuracy.

In the literature, we know only of the studies [3, 4] in regard to investigations of natural convection on a vertical plate in a granular medium under conditions of nonlinear filtration, calculated from the Forscheimer equation

$$u\mu/\Pi + b\rho u^2 = -[\nabla p + \rho g], \tag{1}$$

where μ is dynamic viscosity; ρ is the density of the fluid; b is a proportionality factor; ∇p is the pressure gradient. These two studies employ the same formulation of the problem. The authors showed that similarity solutions do exist in the cases of an isothermal wall [3, 4] and a constant heat flow on the wall [3]. The deviation from the Darcy regime is characterized by the modified Grashof criterion $Gr_b = g\beta\Delta Tb\Pi^2/\nu^2$ (at $Gr_b \to 0$, we have the regime of linear filtration; at $Gr_b \to \infty$, the deviation from Darcy's law is maximal). The results obtained in [3, 4] show that, under conditions of free convection in a porous medium, failure to allow for nonlinear filtration effects leads to a reduction in heat transfer.

Here we examine the problem of heat transfer during mixed convection on a vertical plate placed in a porous medium with a deviation of the filtration regime from Darcy's law.

Formulation of the Problem

The macroscopic equations of mass conservation and heat transfer in granular layers modeling porous media are written as follows [1] for steady-state conditions

$$\partial u/\partial x + \partial v/\partial y = 0; \tag{2}$$

$$u\partial T/\partial x + v\partial T/\partial y = a\partial^2 T/\partial y^2. \tag{3}$$

We take (1) as the equation describing filtrational motion in the region of Reynolds numbers greater than unity. Using the Boussinesq approximation $\rho = \rho_\infty[1 - \beta(T - T_\infty)]$, after cross differentiation we exclude the pressure from (1)

$$\frac{\partial}{\partial y}[f(w^0)u] - \frac{\partial}{\partial x}[f(w^0)v] = \pm\frac{\Pi}{\mu}\rho_\infty g\beta\frac{\partial T}{\partial y}. \tag{4}$$

Here, $f(w^0) = 1 + b\Pi\rho w^0/\mu$; w^0 is the local velocity; the + and — signs correspond to the conditions $T_w > T_\infty$ and $T_w < T_\infty$.

In the general case of power dependences of the wall temperature T_w and the velocity of the external flow U_∞ on the longitudinal coordinate x, the boundary conditions have the form $T = T_w = T_\infty + Ax^\lambda$, $v = 0$ at $y = 0$, $T = T_\infty$, $u = U_\infty = Bx^n$ as $y \to \infty$. The coefficients A and B are assumed to be positive. Having determined the stream function Ψ ($u = \partial\Psi/\partial y$, $v = -\partial\Psi/\partial x$) by the usual method, we formulate the heat-transfer problem as follows in a boundary-layer approximation:

$$\frac{\partial^2\Psi}{\partial y^2} + \frac{b\Pi\rho_\infty}{\mu}\frac{\partial}{\partial y}\left(\frac{\partial\Psi}{\partial y}\right)^2 = \pm\frac{\Pi}{\mu}\rho_\infty g\beta\frac{\partial T}{\partial y}; \tag{5}$$

$$\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}; \tag{6}$$

$$T = T_w = T_\infty + Ax^\lambda,\ \frac{\partial\Psi}{\partial x} = 0 \quad \text{at} \quad y = 0; \tag{7}$$

$$T = T_\infty,\ \frac{\partial\Psi}{\partial y} = Bx^n \quad \text{at} \quad y \to \infty. \tag{8}$$

Let us examine the feasibility of obtaining similarity solutions for Eqs. (5) and (6) with boundary conditions (7) and (8). The form of the similarity variables can be obtained by following the procedure described in [5]: $\eta = \left(\frac{U_\infty}{ax}\right)^{1/2} y$, $\Psi = (aU_\infty x)^{1/2}F(\eta)$, $\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$.

After their insertion into (5)-(8) (the primes denote differentiation), we have

$$F'' + \mathrm{Re}_b (F'^2)' = \pm \frac{\mathrm{Gr}_b}{\mathrm{Re}_b} \Theta'; \quad (9)$$

$$\Theta'' + \frac{n+1}{2} F\Theta' - \lambda F'\Theta = 0; \quad (10)$$

$$\eta = 0,\ \Theta(0) = 1,\ F(0) = 0; \quad (11)$$

$$\eta \to \infty,\ \Theta(\infty) = 0,\ F'(\infty) = 1, \quad (12)$$

where $\mathrm{Re}_b = U_\infty b \Pi/\nu$ is the modified Reynolds number, characterizing the deviation of the filtration regime from Darcy's law; the complex $\mathrm{Gr}_b/\mathrm{Re}_b$ is the parameter of mixed convection.

It is interesting to study the limiting case of maximum deviation from Darcy's law ($\mathrm{Re}_b \to \infty$), this case being the asymptote of the problem being examined. This allows us to establish the main criterional relations for heat transfer in explicit form. These relations can then be used to analyze more complicated situations. Also, such similarity solutions make it possible to easily calculate averaged parameters, which is of particular value in the first stage of engineering calculations.

Boundary Conditions of the First Kind

In the limiting case, when Re_b is sufficiently large, the first term in Eq. (9) can be ignored. Free convection obviously has an effect on motion if $\mathrm{Gr}_b \sim O(\mathrm{Re}_b^2)$.

A similarity solution exists when the complex $\mathrm{Gr}_b/\mathrm{Re}_b^2$ is independent of x, i.e., $\lambda = 2n$. We write the problem in the form

$$(F'^2)' = \pm (\mathrm{Gr}_b/\mathrm{Re}_b^2)\, \Theta'; \quad (13)$$

$$\Theta'' + \frac{1}{4}(n+2) F\Theta' - \lambda F'\Theta = 0 \quad (14)$$

with boundary conditions (11) and (12). Equation (13) can be integrated once, so that with allowance for boundary condition (12), it takes the form

$$(F')^2 = \pm (\mathrm{Gr}_b/\mathrm{Re}_b^2)\, \Theta + 1. \quad (15)$$

Thus, Eqs. (14) and (15), with boundary conditions (11) and (12), describe heat transfer during mixed convection with maximal deviation of the filtration regime from Darcy's law.

It is useful to note the following. The complex $\mathrm{Gr}_b/\mathrm{Re}_b^2$ can be regarded as the specific Froude number for nonisothermal fluid motion modified for the case of nonlinear filtration: $\frac{\mathrm{Gr}_b}{\mathrm{Re}_b^2} = \frac{g\beta\Delta T}{b}\frac{1}{U_\infty^2} = \mathrm{Fr}_b^{-1}$. Thus, Eq. (15) is transformed as follows: $(F')^2 = \mathrm{Fr}_b^{-1}\Theta + 1$.

The local heat flux on the plate

$$q_w = -\Lambda(\partial T/\partial y)_{y=0} = \Lambda \Delta T (U_\infty/ax)^{1/2}[-\Theta'(0)],$$

while the heat-transfer coefficient

$$\alpha = \Lambda (U_\infty/ax)^{1/2}[-\Theta'(0)]$$

or, in dimensionless form

$$\mathrm{Nu}_x/\mathrm{Pe}_x^{1/2} = -\Theta'(0), \quad (16)$$

where $\mathrm{Nu}_x = \alpha x/\Lambda$ is the Nusselt number; $\mathrm{Pe}_x = U_\infty x/a$ is the Peclet number; Λ is the effective thermal conductivity of the filtrating fluid; the values of $[-\Theta'(0)]$ are a function of the complex $\mathrm{Gr}_b/\mathrm{Re}_b^2$.

Boundary Conditions of the Second Kind

In this case, $q_w = -\Lambda(\partial T/\partial y)_{y=0} = \mathrm{const}$. For the characteristic temperature gradient ΔT and the boundary-layer thickness δ, this leads to the estimates $q_w/\Lambda \sim \Delta T/\delta$, $\Delta T \sim q_w\delta/\Lambda$, $\delta \sim \mathrm{Pe}_x^{-1/2}$.

Introducing the similarity variables $\eta = (U_\infty/ax)^{1/2} y$, $\Psi = (aU_\infty x)^{1/2} \cdot F(\eta)$, $\Theta = \frac{T - T_\infty}{q_w}\frac{\Lambda}{x}\mathrm{Pe}_x^{1/2}$, we obtain transformed equations (5) and (6):

$$(F'^2)' = \pm (\mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2})\,\Theta'; \qquad (17)$$

$$\Theta'' + \frac{1}{2}(n+1)F\Theta' - \frac{1}{2}(1-n)F'\Theta = 0 \qquad (18)$$

with the boundary conditions

$$\Theta'(0) = -1,\ F(0) = 0 \quad \text{at} \quad \eta = 0; \qquad (19)$$

$$\Theta(\infty) = 0,\ F'(\infty) = 1 \quad \text{at} \quad \eta \to \infty. \qquad (20)$$

Here, $\mathrm{Ra}_b^* = g\beta q_w x^3/(\Lambda b a^2)$ is the modified Rayleigh number; the complex $\mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2}$ is the parameter of mixed convection. At $n = 1/5$, it is independent of x, and the problem becomes self-similar. Integrating Eq. (17) once with allowance for boundary condition (20), we obtain

$$(F')^2 = \pm (\mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2})\,\Theta + 1. \qquad (21)$$

Thus, similarity problem (18-21) describes heat transfer during mixed convection in a granular bed under conditions of maximal deviation of the filtration regime from Darcy's law with second-kind boundary conditions.

The following relation is valid for the local Nusselt number

$$\mathrm{Nu}_x = \frac{q_w}{T - T_\infty}\frac{x}{\Lambda} = \mathrm{Pe}_x^{1/2}\,[\Theta(0)]^{-1}. \qquad (22)$$

The values of $\Theta(0)$ are a function of the complex $\mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2}$.

Intermediate Regime

When all of the terms in (9) are important, a similarity solution is possible only when $\lambda = n = 0$, i.e., for an isothermal wall and an external flow moving at a constant velocty; here, Eq. (9) retains the same form, while (10) can be simplified:

$$\Theta'' + \frac{1}{2}F\Theta' = 0. \qquad (23)$$

Boundary conditions (11) and (12) are also valid for the problem being examined. The corresponding criteria have the form $\mathrm{Re}_b = Bb\Pi/\nu$, $\mathrm{Gr}_b/\mathrm{Re}_b = g\beta A\Pi/\nu B$. In Eq. (16), for the local Nusselt number, the values of $\Theta'(0)$ will depend on two parameters: Re_b and $\mathrm{Gr}_b/\mathrm{Re}_b$.

Analysis of the Results

The resulting systems of ordinary differential equations (11), (12), (14), (15); (18)-(21); (9), (11), (12), (23) were solved by a numerical method. We used the method of superposition to reduce the boundary-value problem to a Cauchy problem. We then solved the latter by a fourth-order Runge–Kutta method.

Figure 1 shows characteristic profiles of the dimensionless velocity and temperature in the boundary layer with mixed convection under the conditions $T_w > T_\infty$ and $T_w < T_\infty$ for different values of the control parameters G. These profiles are stratified with respect to the values of G. The figure shows cases of first-order boundary conditions for the intermediate regime (a – $\mathrm{Re}_b = 0.10$, $G \equiv \mathrm{Gr}_b/\mathrm{Re}_b$), and for maximal deviation from Darcy's law (b – $\lambda = 1/3$, $G \equiv \mathrm{Gr}_b/\mathrm{Re}_b^2$), as well as second-order boundary conditions at $\mathrm{Re}_b \to \infty$ (c – $G \equiv \mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2}$); the dashed lines show calculated profiles of the temperature and velocity for oppositely directed flows produced by natural and forced convection ($T_w < T_\infty$); curves 1-4 were constructed for the control parameters 100, 10, 1, and 0.1, respectively. Curves 5 and 6 in Fig. 1a and b were constructed for G = 0.5 and 1.0, while curves 5 and 6 in Fig. 1c were constructed for G = 0.3 and 0.6, respectively.

A change in the criterion G makes it possible to determine the effect of natural convection on motion. Large values of the ratios $\mathrm{Gr}_b/\mathrm{Re}_b$ and $\mathrm{Gr}_b/\mathrm{Re}_b^2$ indicate that the effect of natural convection is predominant. Natural convection in the opposite direction, with an increase in G, increases the effect of stagnation phenomena on motion. The profiles of dimensionless excess temperature are analogous in form to the profiles for free or forced convection. We might also note the similar character of the temperature profiles throughout the investigated ranges of the parameters $\mathrm{Gr}_b/\mathrm{Re}_b$, $\mathrm{Gr}_b/\mathrm{Re}_b^2$, $\mathrm{Ra}_b^*/\mathrm{Pe}_x^{5/2}$.

Figure 2 characterizes the change in the dimensionless heat-transfer coefficient $\mathrm{Nu}_x/\mathrm{Pe}_x^{1/2}$ obtained from Eqs. (16) and (22) in relation to the corresponding control parameters G for each of the following cases: a – intermediate regime, when the deviation from Darcy's law is small, with first-kind boundary conditions; b – first-kind boundary conditions with a maximum deviation from the Darcy regime; c – second-kind boundary conditions (constant heat flux on the wall), with $Re_b \to \infty$.

Curves 1-3 in Fig. 2a are constructed for $Re_b = 0.10$, 0.05, and 0 (i.e., for linear filtration in accordance with Darcy's law), while the curves in Fig. 2b were constructed for $\lambda = 1$, 1/3, and 0, respectively. An asterisk is used in Figs. 2a and b to denote calculations performed for oppositely directly flows. Analogous data is represented in Fig. 2c by the short branch of the curve, which indicates a reduction in heat transfer with an increase in the parameter $Ra_b^*/Pe_x^{5/2}$.

The relation $Nu_x/Ra_b^{1/4} = -\Theta'(0)$ was obtained in [3] for the heat-transfer coefficient for free convection on an isothermal wall with maximal deviation from the Darcy regime and $T_w > T_\infty$. This relation can be represented in the form $Nu_x/Pe_x^{1/2} = (Gr_b/Re_b^2)^{1/4}[-\Theta'(0)]$, where $\Theta'(0) = -0.494$ and serves as the asymptote of the solution we obtained when free convection is predominant at $\lambda = 0$ (this case is represented by the dot-dash line in Fig. 2b).

The asymptote for forced convection (the dashed line in Fig. 2b) can be obtained by solving Eqs. (13) and (14) for an isothermal wall ($\lambda = 0$) at $Gr_b/Re_b^2 = 0$. The system takes the form $(F')^2 = 1$, $\Theta'' + F\Theta'/2 = 0$. From this, with allowance for boundary conditions (11) and (12), we find that $\Theta = -\frac{1}{\sqrt{\pi}} \times \int_0^\eta \exp(-\eta^2/2)\, d\eta + 1$. Accordingly, $\Theta'(0) = -1/\sqrt{\pi} = -0.5642$.

In regard to the ranges of the parameter Gr_b/Re_b^2, we can distinguish the following regimes (with an error no larger than 5%) for an isothermal wall: $0 < Gr_b/Re_b^2 < 0.4$ – forced convection; $0.4 < Gr_b/Re_b^2 < 7$ – mixed convection; $Gr_b/Re_b^2 > 7$ – free convection.

For second-kind boundary conditions in the case of maximal deviation from the Darcy regime (Fig. 2c), the asymptote corresponding to the predominance of free convection [3] can be represented in the form (dot-dash line) $Nu_x/Pe_x^{1/2} = [\Theta(0)]^{-1}(Ra_b^*/Pe_x^{5/2})^{1/5}$. The calculations in [3] gave the value $\Theta(0) = 1.243$.

In the case of the predominance of forced convection, the solution of system (18)-(21) at $Ra_b^*/Pe_x^{5/2} \to 0$ leads to the heat-transfer law (dashed line) $Nu_x/Pe_x^{1/2} = 0.8641$. For practical calculations, it is possible to recommend the following heat-transfer regimes with respect to the parameter $Ra_b^*/Pe_x^{5/2}$: $0 < Ra_b^*/Pe_x^{5/2} < 0.35$ – forced convection; $0.35 < Ra_b^*/Pe_x^{5/2} < 5$ – mixed convection; $Ra_b^*/Pe_x^{5/2} > 5$ – free convection.

As shown by analysis of the completed calculations of heat transfer in the intermediate regime (when all of the terms in equation of motion (9) are considered), with small values of the control parameter Gr_b/Re_b, there is almost no stratification of the heat-transfer coefficient with respect to the number Re_b (Fig. 2a). Thus, in the range $0 < Gr_b/Re_b < 0.2$, heat transfer can be calculated on the basis of the asymptote for forced convection (the dashed line in Fig. 2a): $Nu_x/Pe_x^{1/2} = 0.5642$. With an increase in Gr_b/Re_b, stratification with respect to the number Re_b becomes significant; calculations for $Re_b = 0.05$ and 0.10 are shown in Fig. 2a. The dot-dash line shows the asymptote corresponding to free convection in the Darcy filtration regime [1]

$$Nu_x/Pe_x^{1/2} = 0.444\,(Gr_b/Re_b)^{1/2}.$$

At small values of Re_b (in the present case, even at $Re_b \le 0.05$), heat transfer can be calculated to within 5% on the basis of free convection with linear filtration beginning at $Gr_b/Re_b \ge 4.5$.

The effect of deviation of the filtration regime from Darcy's law on heat transfer in mixed convection is shown in Fig. 3, where $\Theta_0'(0)$ is heat transfer in the Darcy regime [6], and $\Theta'(0)$ is our estimate for the intermediate regime with $Re_b = 0.10$.

Figure 4 shows the change in the dimensionless thickness of the boundary layer η_δ in the three types of problems examined above in relation to the corresponding control parameters G: 1) $G \equiv Gr_b/Re_b$, 2) $G \equiv Gr_b/Re_b^2$, 3) $G \equiv Ra_b^*/Pe_x^{5/2}$.

Thus, the completed analysis shows that heat transfer by mixed convection on a vertical plate immersed in a granular bed, with allowance for the contribution of the inertial term in the resistance law, is determined by the ratio of two criteria (first-kind boundary conditions): Gr_b (introduced in [3, 4]) and Re_b.

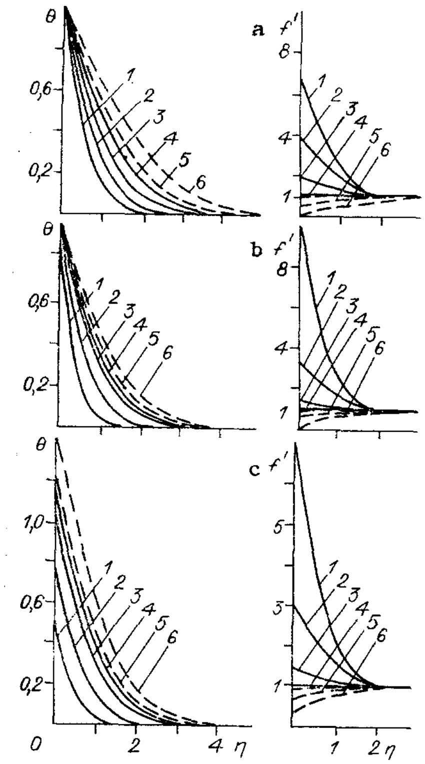

Fig. 1

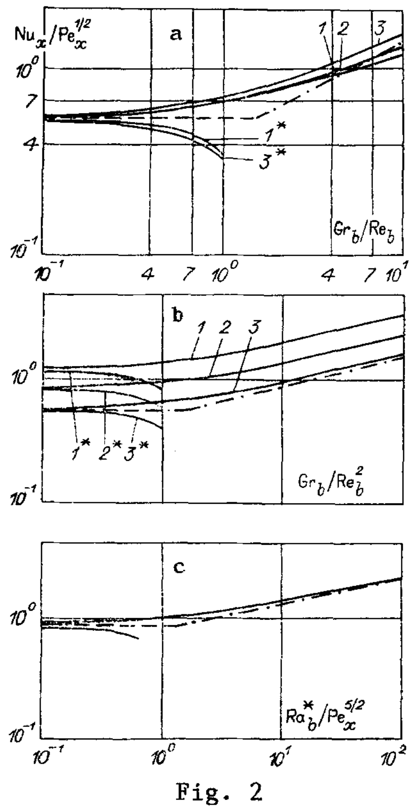

Fig. 2

For second-kind boundary conditions, the characteristic parameter is composed of the criteria Ra_b^* and Pe_x (it is not hard to show that $Ra_b^*/Pe_x^{5/2} \sim Gr_b/Re_b^2$).

We should also point out the following feature of the class of problems examined here: in the case of heat transfer by mixed convection through the motion of a one-phase fluid about a smooth plate, the only convection parameter is the ratio Gr/Re^2 [5]. For a plate in a granular bed with maximal deviation from Darcy's law, we have the control parameter

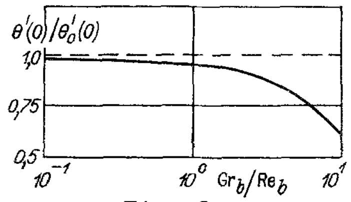

Fig. 3

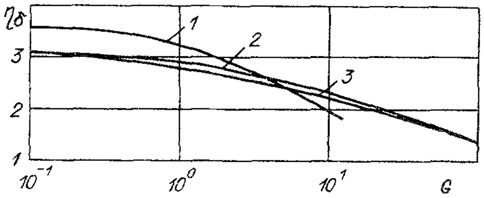

Fig. 4

Gr_b/Re_b^2. In the case where the contributions of the linear and quadratic terms in Eq. (1) are comparable, the control parameter is Gr_b/Re_b. It should also be noted that for granular beds, the numerical values of Gr/Re (the control parameter for a linear filtration law, $Gr = g\beta\Delta T\Pi x/\nu^2$, $Re = u\sqrt{\Pi}/\nu$) and Gr_b/Re_b (with a deviation from the linear law) are identically equal. As noted above, this makes it possible to use the asymptote for free convection, which is analogous to the asymptote for mixed convection under conditions of linear filtration.

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MELTING OF LEAD IN SHOCK COMPRESSION

Yu. V. Bat'kov, V. N. German,
R. S. Osipov, S. A. Novikov,
and V. A. Tsyganov

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The transition of a solid in shock-wave compression to the liquid phase (as in any phase transition during shock compression) occurs within a certain pressure range associated with a section on the Hugoniot curve corresponding to a mixture of two phases. Schemes for the formation of the accompanying flows were examined, for example, in [1, 2] for a phase transformation involving a reduction in volume and in [3] for fusion. It was noted in [3] that the fusion of a substance in a shock wave (SW) cannot be recorded by presently known empirical methods based on the measurement of wave and mass velocities because the change in the parameters of the substance during melting is very small. In well-known experiments, conclusions on the occurrence of melting in an SW were made on the basis of changes in the viscosity of metals behind the shock front [4] and impulsive x-ray diffraction study of the character of motion of the free surface of a specimen during impact [3].

Here, we present experimental results on the fusion of lead in an SW obtained by two independent methods: study of the dependence of the dynamic yield point Y_d on the amplitude of the stress σ_x in unidimensional shock-wave compression; study of microstructural changes in specimens after shock-wave loading.

We used manganin wire stress gauges located in two mutually perpendicular sections of the test specimen to directly measure the longitudinal σ_x and transverse σ_y components of

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